

**More Babies, More War:
The Vicious Cycle of Development, Demography and Conflict**

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Abstract

This paper analyzes the vicious cycle between demography and conflict. It develops a new theoretical framework in which fertility rates and the probability of conflict are endogenously determined, giving rise to multiple equilibria.

The central argument is that high fertility rates produce large cohorts of young adults, increasing the probability of conflict through the *youth bulge* phenomenon. Conflict, in turn, raises the risk of mortality among young adults, inducing families to increase fertility as a form of demographic insurance.

These two mechanisms reinforce one another, generating two stable equilibria: one characterized by high fertility, a young population, and a high probability of conflict; the other by low fertility, an aging population, and a low probability of conflict.

The empirical evidence illustrates the importance of this relationship. Among countries with fertility rates above five children per woman, approximately 75 percent experienced conflict, compared with less than 8 percent among countries with fertility rates below two.

The model offers an explanation for persistent regional differences in demography, conflict, and economic development. While parts of Europe and East Asia have experienced declining fertility, population aging, and relatively low levels of internal conflict, parts of Africa and the Middle East continue to combine high fertility, young populations, and recurrent violence.

The paper shows how the interaction between fertility and conflict can trap economies in a self-reinforcing equilibrium of high population growth, recurrent conflict, and low economic development.

Keywords: fertility rates, youth bulge, demographic transition, conflict, multiple equilibria, economic development.

JEL classification: D74; J13; J14; O40; O11.

I. Introduction

Why do some countries remain trapped in low development for long periods of time, while others move toward sustained economic growth? Traditional theories of development have emphasized capital accumulation, human capital, institutions, technological change, and demographic transition. This paper focuses on another mechanism that has received considerably less attention: the interaction between demography and conflict.

The central argument of the paper is that high fertility and conflict should not be understood as two independent characteristics of poor economies, nor should the relationship between them be viewed as running in only one direction. Instead, fertility and conflict can reinforce one another. High fertility produces young populations and large youth cohorts, increasing the probability of political violence and conflict. Conflict, in turn, can induce families to have more children. The interaction between these two mechanisms creates a vicious cycle of demography and war, which can keep countries in an equilibrium characterized by high fertility, recurrent conflict, low capital accumulation, and little economic growth.

An important distinction concerns the type of conflict analyzed. This paper focuses on organized violence involving terrorist organizations, criminal organizations, gangs, insurgent groups, and military factions rebelling against the authorities. It does not seek to explain wars between sovereign states or major powers. The mechanisms considered here concern recruitment into violent organizations, the expected returns from participation, and the response of the authorities.

The paper is theoretical. It does not attempt to estimate a new causal relationship between fertility and conflict. Rather, the model is built upon a set of well-established empirical regularities documented in the demographic and conflict literature, in particular the work of Cincotta et al. (2003), Urdal (2006), and subsequent research on the “youth bulge.” This literature identifies a strong association between the age structure of populations and the incidence of political violence. Countries with relatively large cohorts of young adults are more exposed to civil unrest and organized conflict, especially when economic opportunities are limited.

Table 1 summarizes this empirical background. The first part reports the relationship between birth rates and the probability of conflict over several historical periods. The probability of conflict rises systematically with the birth rate. The second part reports the same relationship using fertility rates. Among countries with fertility rates of five children per woman or more, 24 of 32 experienced conflict, corresponding to a probability of 75 percent. Among countries with fertility below two children per woman, only 6 of 99 experienced conflict, corresponding to a probability of approximately 6 percent. The intermediate groups show the same pattern: conflict

incidence is 45 percent for fertility between four and five, 24 percent between three and four, and 18 percent between two and three.

These empirical regularities provide the starting point of the model, but they do not explain why countries can remain in such situations for very long periods. The contribution of this paper is to show that the relationship between fertility and conflict may be 'bidirectional' and self-reinforcing.

The first direction of causality runs from fertility to conflict. High fertility today produces a large cohort of young people approximately fifteen to twenty years later. A "youth bulge" is generally defined as a large share of individuals aged 15–29 relative to the adult population. The literature has proposed several mechanisms linking such demographic structures to political violence. Young populations are more willing to engage in risky behavior; large youth cohorts also face greater competition for employment and resources. These effects are particularly strong when economic opportunities are scarce.

The economic mechanism reinforces the demographic one. In the standard Solow framework, rapid population growth reduces capital per capita unless investment increases sufficiently to compensate for population growth. High fertility therefore contributes to low capital intensity, inadequate infrastructure, low wages, and unemployment. Large cohorts of young individuals enter economies that often cannot provide sufficient productive employment. The opportunity cost of participating in gangs, insurgencies, or other violent organizations consequently falls.

Part I formalizes this argument using a framework inspired by Becker's economic analysis of crime. Individuals compare the expected returns from legal employment and participation in organized violence. The size of the young cohort can improve recruitment and organizational success, while repression by the authorities reduces the expected return from violence and raises participation costs. Repression is assumed to intensify as the cohort expands. Over the demographic range considered in the model, the recruitment effect dominates, so the probability of conflict increases with fertility, although at a diminishing rate at higher fertility levels.

The second direction runs from conflict to fertility. This is the mechanism that transforms a demographic explanation of conflict into a theory of a vicious cycle. Part II uses a simple intergenerational model of household decisions, drawing on Becker's economic theory of fertility. Parents derive utility from their own consumption and from their children, while taking account of the risk that some children will die as young adults. When the probability of conflict rises, the anticipated risk of losing a child can increase the incentive to have additional children. Fertility then serves partly as demographic insurance. Under the specified household utility function, the optimal number of children increases linearly with the probability of conflict.

The central step is to combine these two relationships. Fertility affects the probability of future conflict through the youth bulge, while the probability of conflict affects

desired fertility. Both variables are therefore determined endogenously. Building on the multiple-equilibrium models of Krugman (1991), Brezis and Krugman (1996) and Bénassy and Brezis (2013), Part III establishes the conditions under which the two relationships admit three equilibria.

Graph 3 illustrates these equilibria. The intersection of the two curves produces three equilibria. Two are stable and one is unstable. The first stable equilibrium is characterized by low fertility and low conflict. The second is characterized by high fertility and high conflict. The intermediate equilibrium is unstable. Small movements away from it cause the economy to converge toward one of the two stable equilibria. Thus, countries with apparently similar initial conditions may ultimately follow very different demographic and development paths, because demographic choices and conflict risk reinforce each other.

This relation between fertility rates and conflict can explain persistent differences in economic development. In the favorable equilibrium, low fertility permits higher capital accumulation per capita and greater investment in the education and health of each child. The population gradually ages, the youth bulge disappears, and the probability of conflict declines. Peace, in turn, permits further investment in physical and human capital. The economy can enter a virtuous circle of demographic transition, stability, and economic growth.

The unfavorable equilibrium works in exactly the opposite direction. High fertility dilutes capital and family resources and generates large youth cohorts. Weak capital accumulation and insufficient infrastructure contribute to low wages and unemployment. These conditions make recruitment into violent organizations more attractive. Conflict then reinforces the incentives for high fertility, reproducing the demographic conditions that contributed to conflict in the first place. The economy may remain trapped in a combination of demographic pressure, recurrent violence, and low development.

This mechanism may help explain persistent regional differences. Parts of Europe and East Asia have experienced declining fertility, population aging, and relatively low levels of internal conflict. Parts of Sub-Saharan Africa and the Middle East continue to combine high fertility, young populations, and recurrent organized violence. The argument is not that demography alone determines conflict. Institutions, social norms, natural resources, international intervention, and historical circumstances also matter. Rather, the paper identifies a channel through which demographic and political conditions can reinforce one another.

The historical implication is that demographic transition need not be merely a consequence of development. It can also be a condition that makes sustained development easier to achieve. When conflict and fertility reinforce one another, a country may fail to accumulate capital and human capital not simply because fertility is

high or because violence destroys resources, but because each force helps reproduce the other.

The main contribution is therefore not another demonstration that young populations are associated with conflict. It is to close the circle by combining the effect of fertility on organized conflict with an endogenous fertility response to conflict. The resulting multiple equilibria provide a theoretical explanation for the persistence of sharply different demographic and development paths.

This paper provides a simple theoretical explanation for why some countries can remain for long periods in a state of high fertility, recurrent conflict, low capital accumulation, and little economic growth, while others converge toward low fertility, peace, and sustained development.

The paper is structured as follows: Section II reviews the relevant literature, Section III introduces the model, and Section IV concludes.

II. Related Literature

The paper brings together several strands of literature: the economic theory of fertility and family size, the relationship between demographic structure and organized violence, and the role of child mortality and uncertainty in fertility decisions. The central contribution is to connect the two directions of causality. A large young cohort can raise the risk of conflict, while anticipated conflict-related mortality can encourage higher fertility. The interaction may then generate multiple equilibria.

2.1. Fertility decisions, the sibship size effect, and the precautionary demographic motive

The economic analysis of fertility begins with Becker (1960), who treats children as a source of parental utility and fertility as a household choice subject to resource constraints. Becker and Lewis (1973) subsequently emphasize the interaction between the number of children and investment in their quality. A larger family may increase parental satisfaction, but it also requires resources that could otherwise be devoted to each child. This trade-off is central to the model developed in this paper.

Regarding the medical literature, health externalities constitute an important channel through which sibship size may affect children's well-being. Desai (1995) emphasizes the negative health consequences of crowding and greater exposure to infectious diseases, such as measles, chickenpox, and diarrhea. Aaby (1988) and Aaby et al. (1984) examine the relationship between family size, exposure to infection, and child health in developing countries. Larger families may increase children's exposure to infectious diseases and, in some circumstances, the severity of illness. These effects can have long-term consequences for health and human capital.

Another potential mechanism concerns maternal health and the resources available for childcare. Repeated pregnancies, maternal health problems, and the demands of caring for many children may reduce the time and attention available to each child. The psychological and sociological literature also emphasizes the relationship between family size and children's emotional and intellectual development.

These mechanisms are often described as the "sibship size effect" or "resource dilution theory." As the number of children increases, parental income, time, and attention must be divided among more siblings. Guo and VanWey (1999) examine the relationship between sibship size and intellectual development, while Brezis and Ferreira (2016) analyze the sibship size effect within an economic model of fertility. Although the magnitude and causal interpretation of these effects may vary across societies, the literature identifies important costs associated with large families that are not fully captured by the standard benefits of having additional children.

A related question is why parents may choose to have more children when the probability of child mortality increases. This is the precautionary demographic motive. If parents care about the number of children who survive to adulthood, uncertain survival can create an incentive to have additional births. Kalemli-Ozcan (2003) develops an economic explanation of the relationship between mortality risk, fertility, and human capital. The demographic transition literature also examines how declining mortality can reduce the need for precautionary births and encourage greater investment in each child.

This precautionary motive is particularly relevant to conflict. When parents anticipate that conflict will increase mortality among young adults, they may increase fertility to protect the expected continuity of the family. This mechanism does not imply that conflict must increase fertility in every society, since displacement, poverty, family separation, and reduced access to health services may work in the opposite direction.

2.2. Demographic structure and the probability of organized conflict: The Youth Bulge Theory

The second strand of literature studies the relationship between the age structure of populations and political violence. *Youth bulges* are typically defined as large cohorts of individuals aged 15–29 relative to the adult population aged 15 and above. Some studies use the narrower age range of 15–24. High fertility rates, with a generational lag, produce large cohorts of young adults and may increase their share in the adult population.

An extensive sociological, demographic, and economic literature links youth bulges to crime, political instability, and violent conflict. One of the leading scholars in this field, Urdal (2006), examines the relationship between youth bulges and political violence. His econometric analysis provides evidence that large youth cohorts increase

the risk of armed conflict, particularly when economic and institutional conditions are unfavorable. Similarly, Cincotta et al. (2003) document the association between youthful age structures and political instability, while Cincotta and Weber (2021) examine the relationship between population age structures and revolutionary and separatist conflicts.

Brooks et al. (2018) extend the demographic argument to interstate conflict. They find that youthful societies are more conflict prone, whereas aging societies tend to be more peaceful. Although their analysis concerns a different category of conflict from the organized internal violence studied in this paper, it reinforces the broader argument that demographic age structures can influence political and military behavior.

What are the main mechanisms explaining the relationship between youth bulges and conflict? The literature emphasizes two broad sets of factors.

The first concerns biological, psychological, and physical characteristics. Young men are disproportionately represented among participants in violent activities. Goldstein (2004) discusses the relationship between gender, age, and violence, while Mesquida and Wiener (1996, 1999) examine the relationship between male age composition and collective aggression. Neumayer (2003) studies the economic and institutional determinants of violent crime. Together, these studies suggest that the demographic composition of a population may influence the potential supply of participants in violent activities. However, age and biological characteristics alone cannot explain why violence occurs in some societies and not in others.

The second set of factors is socioeconomic. High fertility rates produce large cohorts of young people who eventually enter the labor market. In the Solow framework, rapid population growth reduces capital per worker unless investment expands sufficiently. When investment in physical capital, infrastructure, and education fails to keep pace with population growth, the consequences may include low wages, unemployment, and limited economic opportunities.

These economic conditions are particularly important for understanding recruitment into violent organizations. When young adults have few opportunities for productive employment, the opportunity cost of participating in rebellions, gangs, or militant organizations declines. Collier (2000) emphasizes the economic incentives underlying rebellion, while Collier and Hoeffler (1998, 2004) examine the economic determinants of civil war. Education and employment opportunities may reduce the attractiveness of violent participation by increasing the returns from legal activities.

Choucri (1974) also emphasizes the relationship between demographic pressure, urbanization, and political instability. Large cohorts of unemployed young adults concentrated in urban areas may contribute to social unrest, particularly when infrastructure and public services are inadequate. These pressures do not necessarily

produce violence, but they may facilitate recruitment and collective mobilization when political and economic conditions are unfavorable.

The relationship between youth bulges and political violence is not unique to the modern world. Goldstone (1991, 2002) emphasizes the role of demographic change in historical revolutions and political crises, including the English Revolution and the European revolutions of the nineteenth century. Möller (1968) also emphasizes the political significance of large youth cohorts and examines their relationship with periods of social instability.

The youth bulge argument has also been applied to political instability in the Middle East and North Africa. Urdal (2006) discusses the relationship between youthful demographic structures and political violence, while Zakaria (2001) emphasizes the political consequences of large populations of young people facing limited economic opportunities. Such demographic pressures have frequently been invoked in discussions of political radicalization and unrest in the region.

Finally, the relationship between population pressure and economic conditions has a long intellectual history. Malthus emphasized the tension between population growth and available resources, while Marx and Engels approached population and poverty through the organization of production and social relations. These perspectives differ substantially, but both contributed to the debate on the economic consequences of demographic change.

The present paper builds on this literature by focusing on a specific mechanism: large cohorts of young adults can increase recruitment opportunities for organized violence, while economic conditions and repression influence the incentives to participate. The contribution of the model is not simply to establish a relationship between youth bulges and conflict, but to combine this relationship with the reverse effect of anticipated conflict on fertility decisions.

2.3. Population pressure, development, and the feedback mechanism

The relationship between population growth and economic development has been extensively analyzed in the economic literature. Galor and Weil (2000) connect demographic transition and economic development in a unified growth framework, emphasizing the interactions between population growth, technological progress, and human capital accumulation.

However, the literature on demographic transition and economic growth does not generally incorporate the reciprocal relationship between fertility and organized conflict. Similarly, while the youth bulge literature emphasizes the effects of demographic structure on political violence, the literature on mortality and fertility focuses on household responses to survival risks. The contribution of the present paper is to bring these mechanisms together and show how their interaction can generate

multiple equilibria, including a persistent equilibrium characterized by high fertility and a high probability of conflict.

2.4. Empirical patterns

Tables 1 and 2 summarize the descriptive evidence and demographic indicators used to motivate the analysis. Table 1 reports a strong association between fertility and the incidence of conflict: among countries with fertility of five children per woman or more, 24 of 32 experienced conflict, compared with 6 of 99 among countries with fertility below two. These are descriptive associations, not estimates of a causal effect. Table 2 illustrates the differences in fertility, median age, birth rates, and the size of young cohorts across countries. The theoretical model examines a possible mechanism behind these patterns, rather than treating them as proof of its assumptions.

III. The model

The model describes the interaction between the number of children per family, n , and the probability of conflict, $\pi \leq 1$. The multiple equilibria model is based on two equations that define the relationship between these two endogenous variables.

As emphasized in the introduction, the paper focuses on organized violence involving terrorist organizations, criminal organizations, gangs and military factions that rebel against the authorities. It does not analyze wars between major powers. We denote the probability of conflict by π , recalling the Greek word *polemos* (πόλεμος), meaning war.

Part I derives the probability of conflict from the fertility rates, the size of the young cohort and the economic incentives for participation in violence. Part II derives fertility from parental maximization. Part III combines the two relationships and examines the conditions for one or three equilibria.

Both relationships build on Gary Becker's economic approach. Part I, examining how fertility affects conflict, draws on Becker's (1968) analysis of crime and punishment. Part II, examining how conflict affects fertility, draws on Becker's (1960) economic theory of fertility. We begin with the first relationship.

Part I. The effect of fertility on the probability of conflict

The starting point is Becker's (1968) economic approach to crime: an individual compares the expected returns from alternative activities, including the costs of punishment. We apply this logic to recruitment into organized violence. We first analyze the probability of organizational success conditional on action, q , and then the probability that conflict occurs, π .

The important link to fertility rates is that higher fertility rates produce a larger young cohort approximately twenty years later, expanding the potential recruitment pool for violent organizations. A larger pool makes recruitment and coordination easier and can increase the expected return from organized violence. The resulting incentive to mobilize also depends on legal employment opportunities and on repression by the political authorities. Let us start with the relationship between fertility rates and the young bulge cohort.

1.1 Fertility and the youth bulge

Let Ω_t denote the size of the young adult cohort relevant to recruitment into violent organization, and n_{t-1} the fertility rate in the preceding generation. In a model with two generations, assume

$$\Omega_t = \beta n_{t-1}, \quad \beta > 0. \quad (1)$$

The coefficient β converts fertility into the size of the relevant cohort, holding other demographic determinants fixed. In a stationary equilibrium, $\Omega = \beta n$. Thus, Ω measures the size of the young cohort. Urdal (2006) studies the association between youth bulges and political violence, emphasizing the role of economic and institutional conditions. A larger recruitment pool does not imply that all its members engage in violence.

1.2 Expected returns from violence and the probability of conflict

A potential recruit compares the expected payoff from participating in an armed organization with the income available from legal work. Following Becker (1968), the expected return incorporates the probability of success, the payoff from success, direct participation costs and expected sanctions. Brezis and Verdier (2013) similarly analyze the decision to fight in terms of expected gains and repression.

Let $B > 0$ be the payoff conditional on organizational success, q its probability of success, C_v the cost of participation and w the return from legal work. The net expected advantage of violence over legal work, D , is

$$D = Bq - w - C_v. \quad (2)$$

At the organizational level, conflict occurs when the net advantage is positive. The probability of conflict is therefore

$$\pi = \Pr[D > 0]. \quad (3)$$

The following subsections specify how cohort size and repression affect the probability of success and participation costs.

1.2.1 The probability of success

The probability that a violent organization achieves its objectives, conditional on undertaking action, depends on two elements. A larger recruitment pool, Ω , facilitates coordinated action, whereas repression by the authorities, R , reduces the prospect of success. Thus $q = q(\Omega, R)$, with $\partial q/\partial \Omega > 0$ and $\partial q/\partial R < 0$. For the numerical illustration, we use the linear specification

$$q(\Omega, R) = q_0 + q_1\Omega - dR, \quad q_1 > 0, d > 0. \quad (4)$$

1.2.2 Effective repression

Becker (1968) treats enforcement and punishment as policy choices. Brezis and Verdier (2013) analyze repression as an endogenous choice of the authorities. To capture a response that intensifies as the recruitment pool expands, we assume the quadratic reaction function

$$R(\Omega) = r_0 + r_1\Omega + r_2\Omega^2, \quad (5)$$

where $r_0 \geq 0$, $r_1 \geq 0$ and $r_2 > 0$. This reaction function specifies the authorities' response; we do not derive it from a separate optimization problem here. Substituting equation (5) into equation (4) gives

$$q(\Omega) = (q_0 - dr_0) + (q_1 - dr_1)\Omega - dr_2\Omega^2. \quad (6)$$

An expanding cohort therefore has opposing effects: it facilitates recruitment but also induces greater repression.

If the direct recruitment effect dominates, $q_1 - dr_1 - 2dr_2\Omega > 0$. We restrict attention to a demographic range over which $0 < q(\Omega) < 1$, so that the linear specification remains a valid probability.

1.2.3 Participation costs

Granovetter's (1978) threshold model explains how willingness to participate depends on the participation of others. Kuran (1989) similarly analyzes how visible opposition may generate a cascade of mobilization. These contributions motivate the role of recruitment and coordination costs in organized violence.

Participation costs include recruitment, coordination, exposure to punishment and the risks associated with repression. We assume that a larger cohort lowers recruitment costs, whereas greater repression raises participation costs. Moreover, we add a random component, ε , which captures unobserved differences in the cost of making a violent organization operational:

$$C_v = c_0 - c_m\Omega + c_rR + \varepsilon, \quad (7)$$

where $c_0 \geq 0$, $c_m > 0$ and $c_r > 0$.

1.2.4 The probability of conflict

Equation (3) relates the probability of conflict to the net advantage of violent participation. Substituting equations (4), (5) and (7) into equation (2) gives

$$D(\Omega, \varepsilon) = Bq - w - C_v = A + K\Omega - H\Omega^2 - \varepsilon, \quad (8)$$

where

$$A = Bq_0 - w - c_0 - (Bd + c_r)r_0,$$

$$K = Bq_1 + c_m - (Bd + c_r)r_1,$$

$$H = (Bd + c_r)r_2 > 0.$$

We assume $K > 0$ and focus on the demographic range $K - 2H\Omega > 0$, which we define as Condition I. A larger cohort then increases the net incentive to engage in violence, while its marginal effect declines as repression intensifies.

When the young cohort Ω is small, organizational success is uncertain and recruitment is costly. As the cohort expands, recruitment becomes easier and expected success may rise. The authorities also intensify repression, reducing success relative to what it would otherwise have been and increasing participation costs.

From equations (3) and (8), we obtain that the probability of war is:

$$\pi(\Omega) = \Pr[D(\Omega, \varepsilon) > 0] = \Pr[\varepsilon < A + K\Omega - H\Omega^2]. \quad (9)$$

To obtain a tractable probability specification, we assume that ε follows a logistic distribution, as in McFadden (1974). It translates heterogeneity in net participation costs into a smooth probability of conflict.

$$F_\varepsilon(x) = \frac{1}{1 + \exp(-\lambda x)} \quad \lambda > 0 \quad (10)$$

where $\lambda > 0$. Using $\Omega = \beta n$ in a stationary equilibrium, we obtain:

$$\pi(n) = \frac{1}{1 + \exp\{-\lambda[A + K\beta n - H\beta^2 n^2]\}} \quad (11)$$

Equation (11) is illustrated in Graph 1. Fertility affects the size of the young cohort, which in turn influences the probability of organizational success, participation costs and the response of the authorities. The form of the curve is initially convex and then concave as explained in the next proposition.

Proposition 1.

Under condition I, we obtain that fertility rates raise the probability of conflict. Moreover, under condition II, the curve is initially convex, and then subsequently becomes concave. There is at most one inflection point as shown in Graph 1.

Condition II. $\lambda[1 - 2\pi(0)]K^2 > 2H$

The proof is presented in Appendix 1. The mechanism is economically intuitive. A larger young cohort facilitates recruitment and raises the expected returns from organized violence. At the same time, repression intensifies, eventually reducing the marginal increase in conflict risk. Heterogeneity in organizational costs implies that the relationship between fertility and the probability of conflict is initially convex and subsequently becomes concave.

We now examine the reverse relationship: how the probability of conflict affects household fertility decisions.

Part II. The effect of conflict on fertility

The economic theory of fertility, developed by Becker (1960), provides a framework for analyzing parental decisions. We use a simple two-generation household model, in the spirit of Baland and Robinson (2000), to derive the effect of conflict on fertility rates. We assume an intergenerational model. Adults work, consume, and raise children, taking account of both the cost of children and the risk that some will die as young adults, mainly due to violence and conflicts.

Let π denote the probability of conflict and $\alpha \in (0,1)$ the probability of death conditional on conflict. The probability of conflict-related mortality is $\alpha\pi$, and the expected number of surviving children is

$$N = n(1 - \alpha\pi) \quad (12)$$

A high probability of conflict does not imply a comparably high mortality rate: α and π are separate parameters.

Parents derive utility from their own consumption and from children. The marginal benefit of additional births declines with family size, consistent with a sibship-size effect. We also allow an anticipated replacement or family-continuation motive: expected mortality raises the benefit of another birth.¹

We represent these motives with the utility function

$$V(n, \pi) = U(C_p) + (\theta + \rho\alpha\pi)n - \frac{\kappa}{2}n^2 \quad (13)$$

subject to the household budget constraint

$$C_p = Y - cn \quad (14)$$

¹ See Brezis and Ferreira (2016), on the sibship size effect, and Kalemli-Ozcan (2003) on the precautionary demographic motive.

where Y is parental income; $c > 0$ is the direct cost per child; $\theta > 0$ is the baseline altruistic benefit; $\rho > 0$ measures the precautionary demographic motive; and $\kappa > 0$ measures diminishing marginal benefit and increasing family-size costs. We assume $U(C_p) = C_p$ for tractability. Substitution yields the maximization problem

$$\max_{n \geq 0} V(n, \pi) = Y - cn + (\theta + \rho\alpha\pi)n - \frac{\kappa}{2}n^2 \quad (15)$$

The first-order condition is

$$\frac{\partial V}{\partial n} = -c + \theta + \rho\alpha\pi - \kappa n = 0 \quad (16)$$

Consequently, optimal fertility is

$$n^*(\pi) = \frac{\theta - c}{\kappa} + \frac{\rho\alpha}{\kappa}\pi \quad (17)$$

The second-order condition is $-\kappa < 0$, so the interior optimum is unique. We assume $\theta > c$ and sufficient income to finance the optimal fertility throughout the relevant range. Defining $n_0 = (\theta - c)/\kappa$ and $\gamma = \rho\alpha/\kappa > 0$, we obtain

$$n^*(\pi) = n_0 + \gamma\pi \quad (18)$$

Differentiating,

$$\frac{dn^*}{d\pi} = \gamma > 0, \quad \frac{d^2n^*}{d\pi^2} = 0 \quad (19)$$

We obtain that the relationship between fertility rates and conflict probability is positive and linear, as presented in the next proposition.

Proposition 2.

The optimal fertility rate increases with the probability of conflict. With the specified utility function, the relationship is linear, as shown in graph 2, and its slope depends on the strength of the precautionary demographic motive, ρ and the probability of mortality conditional on conflict, α .

It is important to note that conflict may have two opposing effects on fertility. Greater conflict risk lowers the expected number of surviving children and could reduce the ordinary altruistic return from another birth. At the same time, concern for the continuation of the family line can increase the marginal benefit of an additional birth.

Graph 2 presents the effects of conflict on fertility rates, with fertility on the horizontal axis and conflict probability on the vertical axis; equation (18) can be rewritten as follows:

$$\pi = \frac{n - n_0}{\gamma} \quad (20)$$

We now turn to the determination of the equilibrium.

Part III. Multiple Equilibria

The two endogenous variables are fertility, n , and the probability of conflict, π . An equilibrium is a pair (n^*, π^*) satisfying simultaneously the conflict equation (11) and the fertility equation (18). We therefore solve two equations in two unknowns.

The two equilibrium relationships are:

$$\pi = \frac{1}{1 + \exp\{-\lambda(A + K\beta n - H\beta^2 n^2)\}}, \quad n = n_0 + \gamma\pi. \quad (21)$$

Substituting the fertility equation into the conflict equation, and taking logarithms, gives a single equation in the probability of conflict:

$$\ln\left(\frac{\pi}{1-\pi}\right) = \lambda[A + K\beta(n_0 + \gamma\pi) - H\beta^2(n_0 + \gamma\pi)^2]. \quad (22)$$

Let us analyze the conditions for multiple equilibria.

3.1. Conditions for multiple equilibria

Proposition 3.

If $J(\cdot)=0$ has three distinct solutions, under Condition I, the model displays three equilibria. The low and high equilibria are locally stable, while the intermediate equilibrium is unstable. The function $J(\cdot)$ denotes the difference between the two sides of equation (22).

Proof

First, we maintain Condition I. To identify the number of intersections, let $J(\pi)$ denote the difference between the two sides of equation (22). An equilibrium requires $J(\pi) = 0$:

$$J(\pi) = \ln\left(\frac{\pi}{1-\pi}\right) - \lambda[A + K\beta(n_0 + \gamma\pi) - H\beta^2(n_0 + \gamma\pi)^2]. \quad (23)$$

The derivative of this difference is:

$$J'(\pi) = \frac{1}{\pi(1-\pi)} - \lambda\beta\gamma[K - 2H\beta(n_0 + \gamma\pi)]. \quad (24)$$

Three equilibria can arise when the interaction between fertility and conflict is sufficiently strong and the two equilibrium relationships are appropriately positioned. The precise mathematical conditions, including the turning-point argument and stability condition, are presented in Appendix 2.

The economic mechanism is simple. Conflict encourages additional births, which expand the future recruitment pool and can increase conflict. Whether this feedback

generates one or three equilibria depends both on its strength and on the relative positions of the fertility and conflict curves.

Indeed, at low fertility, an increase in conflict induces some additional births, but the resulting increase in recruitment is too small to sustain a large rise in conflict. The economy returns toward the low-conflict equilibrium. At intermediate fertility, the demographic response is sufficiently strong to reinforce an initial disturbance, so the middle equilibrium is unstable. At high fertility, the response of the authorities and the bounded nature of conflict probabilities reduce the marginal effect of further births; the high-conflict equilibrium is again stable.

3.2. Numerical illustration

To illustrate the mechanism with the actual equations of Parts I and II, we assume

$$A = -5.922347, \quad K = 2, \quad H = 0.13, \quad \beta = \lambda = 1, \quad n_0 = 1.6, \quad \gamma = 5 \quad (25)$$

Thus, the two reactions are:

$$\pi(n) = \frac{1}{1 + \exp(5.922347 - 2n + 0.13n^2)}, \quad n = 1.6 + 5\pi \quad (26)$$

The relevant fertility interval is $1.6 \leq n \leq 6.6$. Throughout this interval, $K - 2Hn > 0$, so the probability of conflict increases with fertility, as required by Condition I.

Solving the two equations gives the following three equilibria:

Equilibrium	Conflict probability π	Fertility n	Slope $G'(\pi)$
Low (stable)	0.0800	2.0000	0.5446
Middle (unstable)	0.4015	3.6073	1.2761
High (stable)	0.7300	5.2498	0.6259

The low and high equilibria have slopes below one, while the middle equilibrium has a slope above one. Graph 3 illustrates these same three equilibria.

3.3 Conclusion of the model

The model formalizes a two-way relationship between demography and organized conflict. The first relationship derives conflict risk from recruitment incentives,

organizational costs and the response of the authorities; the second derives fertility from parental optimization with an anticipated mortality-insurance motive. Together they can generate three stationary equilibria, with two locally stable outcomes separated by an unstable threshold.

Two ingredients are important for a development trap. First, the fertility response to conflict, measured by γ , must be sufficiently strong. Second, the demographic response of conflict, governed by the slope of equation (11), must also be sufficiently strong. The position of the curves also matters: higher legal wages or expected participation costs shift the conflict reaction downward, while changes in baseline fertility shift the fertility reaction. Such changes can eliminate the high-conflict equilibrium even if feedback remains strong.

The model thus provides a mechanism through which demographic pressure and organized violence can perpetuate one another. Its behavioral assumptions and parameter values remain to be tested.

IV. Conclusion

This paper develops a theoretical framework to explain the vicious cycle between demography and conflict. Its central argument is that fertility rates and the probability of conflict are not independent variables, nor does causality run in only one direction. High fertility produces large cohorts of young adults, increasing the potential for recruitment into violent organizations and raising the probability of conflict. Conflict, in turn, increases the risk of mortality among young adults and may induce families to have more children as a form of demographic insurance. These two mechanisms reinforce one another.

The interaction generates multiple equilibria. A country may converge toward a stable equilibrium characterized by low fertility and a low probability of conflict, or remain trapped in another stable equilibrium characterized by high fertility and recurrent violence. Between these two outcomes lies an unstable equilibrium that separates their paths of adjustment. Thus, countries with relatively similar economic conditions may follow very different demographic and development trajectories.

This mechanism has important implications for economic growth. High fertility reduces capital per capita unless investment increases sufficiently, places pressure on infrastructure, and limits the resources available for education and human capital formation. Large cohorts of young adults entering an economy with insufficient employment opportunities can increase the attractiveness of participation in violent organizations. Conflict, in turn, discourages investment, destroys resources, and reinforces the incentives for high fertility. Instead of converging toward the standard development path, an economy may remain trapped in a self-reinforcing equilibrium of high fertility, conflict, and low development.

This interpretation helps explain persistent regional differences. Parts of Europe and East Asia have experienced substantial fertility declines, population aging, high levels of capital per capita, and relatively low probabilities of internal conflict. By contrast, parts of Sub-Saharan Africa and the Middle East continue to combine high fertility, young populations, limited capital accumulation, and recurrent political violence.

Table 1 shows that among countries with fertility rates of five children per woman or more, approximately 75 percent experienced conflict, compared with about 6 percent among countries with fertility rates below two.

An important qualification concerns the nature of conflict. The model focuses on organized violence involving terrorist organizations, criminal organizations, gangs, insurgent groups, and armed factions challenging political authorities. Conflicts involving organizations such as Hezbollah, Hamas, and the Houthis in the Middle East, and the ISIS-DRC, the M23 movement and the FDLR in Africa fall within its scope.

The model does not seek to explain wars between sovereign states or major powers, such as a possible confrontation involving the United States, China, Russia, or the European Union. Decisions to initiate such wars depend on additional considerations, including geopolitical ambitions, strategic interests, and the political objectives of national leaders.

The model deliberately concentrates on two endogenous variables: fertility and the probability of conflict. It does not explicitly incorporate political institutions, religious beliefs, social norms, natural resources, or international intervention. These factors undoubtedly influence demographic decisions and the likelihood of violence. Their omission is not intended to deny their importance, but to isolate a mechanism that has received insufficient attention: the possibility that demographic behavior and conflict reinforce one another and generate persistent differences across societies.

The existence of multiple equilibria also has important policy implications. If the high-fertility, high-conflict outcome is a stable equilibrium, modest improvements in economic conditions may not be sufficient to move an economy out of the trap. Policies that improve employment opportunities for young adults, strengthen education and human capital formation, or reduce the incentives for recruitment into violent organizations can weaken the link between fertility and conflict. Policies that alter the demographic response to insecurity may also affect the equilibrium. Under appropriate conditions, such interventions can eliminate the high-conflict equilibrium or move the economy toward the low-fertility, low-conflict outcome. Identifying the most effective instruments requires extending the model to incorporate explicit policy choices.

The main contribution of this paper is therefore to go beyond the established correlation between population age structure and political violence. The relationship is not simply one in which demography determines conflict. Conflict may itself influence demographic decisions, reproducing the conditions that make future violence more

likely. The development trap is not caused by high fertility alone, or by conflict alone, but by the interaction between the two. Understanding this interaction is essential for explaining why some societies remain caught in persistent cycles of demographic pressure, violence, and low development, while others achieve demographic transition, political stability, and sustained economic growth.

TABLES and FIGURES

TABLE 1

FERTILITY RATES, BIRTH RATES AND PROBABILITY OF CONFLICTS

Birth Rates 1985-1990	Probability of conflict 1971-1980	Probability of conflict 1981-1990	Probability of conflict 1991-2000
45.0 or more	40%	45%	53%
35.0 to 44.9	29%	33%	34%
25.0 to 34.9	22%	31%	24%
15.0 to 24.9	15%	10%	16%
Less than 15.0	2%	2%	5%
Fertility Rates			
2001-2010	2001-2010	# conflicts world	# countries world
5 and more	75%	24	32
4 to 5	45%	10	22
3 to 4	24%	7	29
2 to 3	18%	13	71
0 to 2	6%	6	99
Total		60	253

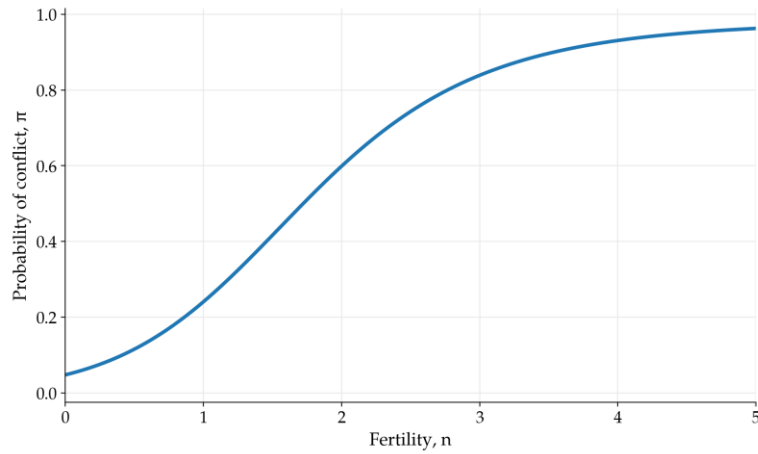
Sources: Cincotta et al., 2003 and own calculations

TABLE 2
Demographic Variables in various countries

	Median Age	Fertility Rate	Birth rate (/1000)	Youth Bulge (%)
Burkina Faso	17.7	4.2	35	55
Gaza	16	5.2	40	49
Mali	15.7	5.6	42	51
Niger	15.6	6.1	45	36
Nigeria	18.1	4.5	37	51
Yemen	18.4	4.6	30	48
Somalia	15.6	6.1	44	51
Greece	46.8	1.3	8	14
Italy	50.0	1.3	7	17
Israel	29.2	2.9	20	30
Japan	49.8	1.2	7	18
US	38.5	1.6	11	20
China	40.1	1.1	7	20
South Korea	44.5	1.1	5	15
France	42.3	1.7	11	21

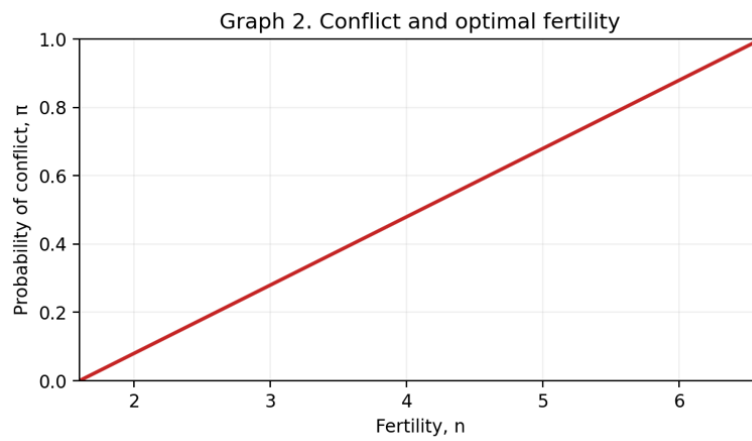
Sources: World Bank and Statista.

Graph 1 The Effects of Fertility on the Probability of Conflict



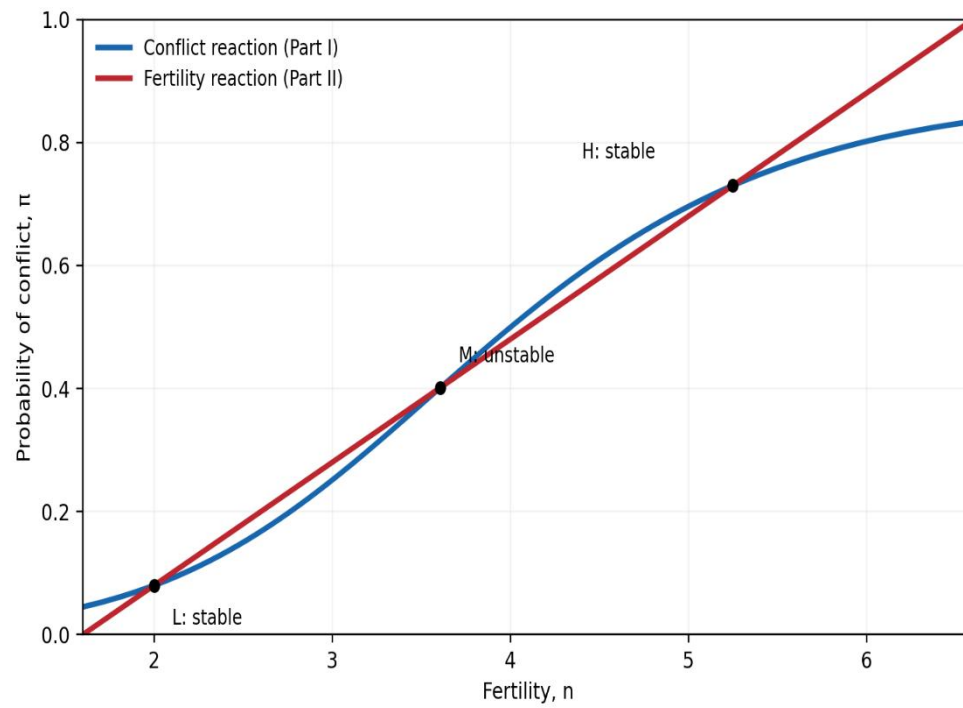
Note: The parameters used are: $A = -3$, $K = 2$, $H = 0.15$ and $\beta = \lambda = 1$. The graph covers $0 \leq n \leq 5$, within the increasing region $n < K/(2H\beta) = 6.67$.

Graph 2: The Effects of the Probability of Conflicts on Fertility Rates



Note: The parameters used are: $n_0 = 1.6$ and $\gamma = 5$; hence $\pi = (n - 1.6)/5$ for $1.6 \leq n \leq 6.6$.

Graph 3: Multiple Equilibria between Fertility rates and Probability of Conflict



Note: Graph 3 is calculated from equations (11) and (20), using the parameter values in equation (25).

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Appendices

Appendix 1. Proof of Proposition 1

Let $X(n) = A + K\beta n - H\beta^2 n^2$. Differentiating equation (11) gives

$$\pi'(n) = \lambda\pi(1 - \pi)\beta(K - 2H\beta n) \quad (\text{A1})$$

which is positive under the stated assumption. Differentiating once more gives

$$\pi''(n) = \lambda\pi(1 - \pi)\beta^2[\lambda(1 - 2\pi)(K - 2H\beta n)^2 - 2H] \quad (\text{A2})$$

The curve is convex when $\lambda(1 - 2\pi)(K - 2H\beta n)^2 > 2H$, and concave when the inequality is reversed. While $\pi < 1/2$, both positive factors, $1 - 2\pi$ and $(K - 2H\beta n)^2$, decrease as n rises within the increasing region. Their product therefore decreases strictly.

Once $\pi \geq 1/2$, the expression in braces is strictly negative. Moreover, it tends to $-2H$ as n approaches $K/(2H\beta)$ from below. Thus, if the initial convexity condition holds and the range extends sufficiently far, the curve changes from convex to concave exactly once. The inflection point occurs below $\pi = 1/2$.

Appendix 2. Proof of Proposition 3

An equilibrium is a solution of $J(\pi) = 0$, where J is defined in equation (23). Since the quadratic expression is finite for $0 \leq \pi \leq 1$, while $\ln[\pi/(1 - \pi)]$ tends to $-\infty$ as π approaches zero and $+\infty$ as π approaches one, at least one equilibrium exists.

The function $J(\pi)$ measures the difference between the two sides of the equilibrium equation. Its zeros are equilibria. Its turning points are different: π_1 is where J reaches a local maximum, and π_2 is where it reaches a local minimum. At those points $J'(\pi) = 0$, but generally $J(\pi) \neq 0$. Figure A1 illustrates the distinction using the numerical parameters of Part III.

Step 1. Number of equilibria.

Differentiating equation (24) gives

$$J''(\pi) = \frac{2\pi - 1}{\pi^2(1 - \pi)^2} + 2\lambda H\beta^2 \gamma^2 \quad (\text{A3})$$

A further derivative yields

$$J'''(\pi) = \frac{2}{(1 - \pi)^3} + \frac{2}{\pi^3} > 0 \quad (\text{A4})$$

Thus J' is strictly convex. It can have at most two distinct zeros in $(0,1)$. Since J' tends to $+\infty$ at both endpoints, if two such zeros $\pi_1 < \pi_2$ exist, J' is positive before π_1 , negative between π_1 and π_2 , and positive after π_2 . Hence J has a local maximum at π_1 and a local minimum at π_2 . There can be at most three distinct equilibria.

On Figure A1, the red dots on the horizontal axis are the three equilibria ($J = 0$). The black dots are the two turning points ($J' = 0$). The maximum must lie above zero and the minimum below zero for three equilibria to exist.

Because J tends to $-\infty$ at zero and $+\infty$ at one, it crosses the horizontal axis three times precisely when the local maximum is positive and the local minimum is negative, provided that J' has two distinct zeros, the necessary and sufficient conditions for three equilibria are:

$$J(\pi_1) > 0 \quad \text{and} \quad J(\pi_2) < 0. \quad (\text{A5})$$

If either expression equals zero, one crossing becomes a tangency, giving two distinct equilibria. If the strict inequalities do not hold and there is no tangency, there is only one equilibrium.

Step 2. Local stability.

Stability requires an adjustment rule. Consider the generational adjustment in which anticipated conflict in one generation determines fertility, which in turn determines conflict in the next:

$$\pi_{t+1} = G(\pi_t) \quad G(\pi) = \frac{1}{1 + \exp[-\lambda X(n_0 + \gamma\pi)]} \quad (\text{A6})$$

where $X(n) = A + K\beta n - H\beta^2 n^2$

At an equilibrium, $G(\pi^*) = \pi^*$. Differentiation gives

$$G'(\pi^*) = \lambda\beta\gamma[K - 2H\beta(n_0 + \gamma\pi^*)]\pi^*(1 - \pi^*) \quad (\text{A7})$$

Combining this expression with equation (24), evaluated at an equilibrium, yields

$$G'(\pi^*) = 1 - \pi^*(1 - \pi^*)J'(\pi^*) \quad (\text{A8})$$

Under Condition I, $G'(\pi^*) > 0$. Therefore, at the first and third crossings $J'(\pi^*) > 0$, so

$0 < G'(\pi^*) < 1$ and these equilibria are locally stable.

At the middle crossing $J'(\pi^*) < 0$, so $G'(\pi^*) > 1$ and the middle equilibrium is unstable.

QED

Figure A1. Equilibria and turning points of $J(\pi)$ 